How classical spacetimes could emerge from quantum gravity

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Outline

Seminal paper: Ashtekar, Kaminski and Lewandowski [0901.0933].

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Formally quantize the system (in *harmonic time*, τ, defined by $N = a^3$):

$$\hat{H} = \hat{H}_o \otimes \hat{I} + \frac{1}{2} \sum_k \left[\hat{I} \otimes \hat{\pi}_k^2 + k^2 \hat{a}^4 \otimes \hat{\phi}_k^2\right]$$

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Approx: no entanglement between geometry ($\Psi_o \in \mathcal{H}_{\text{geom}}$) and matter ($\varphi \in \mathcal{H}_{\text{matt}}$)

$$\Psi(\tau) = \Psi_o(\tau) \otimes \varphi(\tau), \qquad i\frac{d}{d\tau}|\Psi_o\rangle = \hat{H}_o|\Psi_o\rangle$$

QFT on quantum spacetime:

$$i\frac{d}{d\tau}|\varphi\rangle = \frac{1}{2}\sum_k \left[\hat{\pi}_k^2 + \langle\Psi_o|\hat{a}^4|\Psi_o\rangle k^2\hat{\phi}_k^2\right]|\varphi\rangle$$

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QFT on effective spacetime:

$$i\frac{d}{dt}|\varphi\rangle = \frac{1}{2}\sum_k \left[\frac{N}{a^3}\hat{\pi}_k^2 + Nak^2\hat{\phi}_k^2\right]|\varphi\rangle$$

QFT on quantum spacetime:

$$i\frac{d}{d\tau}|\varphi\rangle = \frac{1}{2}\sum_k \left[1\hat{\pi}_k^2 + \langle\Psi_o|\hat{a}^4|\Psi_o\rangle k^2\hat{\phi}_k^2\right]|\varphi\rangle$$

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whose unique solution is

$$N(\tau) = a(\tau)^3, \qquad a(\tau) = \langle\Psi_o(\tau)|\hat{a}^4|\Psi_o(\tau)\rangle^{\frac{1}{4}}$$

Interpretation: the dynamics of a massless quantum field ϕ on quantum spacetime Ψ_o is equivalent to the dynamics of ϕ on effective spacetime

$$ds^2 = -N^2 dt^2 + a^2 d\vec{x}^2 = -\langle \hat{a}^4 \rangle^{\frac{3}{2}} d\tau^2 + \langle \hat{a}^4 \rangle^{\frac{1}{2}} d\vec{x}^2$$

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Can we generalize?

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A simple generalization, yet enough to:

- bring up deformations of dispersion relations
- have application in physical cosmology

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System of 3 equations for unknowns N and a:

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Two approaches:

1. make the effective mass an unknown (Agullo, Ashtekar, Neslon [1211.1354])
2. put together second and third eq's (Assanioussi, AD, Lewandowski [1412.6000])

approach 1: 3 equations for 3 unknowns

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whose unique solution is: a and N as in the massless case, moreover

$$\bar{m} = \frac{\langle \hat{a}^6 \rangle^{\frac{1}{2}}}{\langle \hat{a}^4 \rangle^{\frac{3}{4}}} m$$

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$\Rightarrow$ Effective metric depends on the wavenumber k/m of the mode considered!

Interpretation: the dynamics of quantum field ϕ of mass *m* on quantum spacetime Ψ_o is equivalent to dynamics of the *same* field on a 1-parameter family of effective spacetimes

$$ds^2_{(k)} = -N_k^2 d\tau^2 + a_k^2 d\vec{x}^2, \qquad N_k = a_k^3$$

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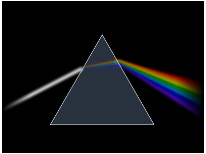

$g^{(k)}_{\mu\nu}$ is a *rainbow metric*, and thus presents *modified dispersion relation*

dispersion relation

Identify the frame of a *classical observer*: $\{u^\mu, e_i^\mu\}$ s.t.

$$u \cdot u = -1, \qquad u \cdot e_i = 0, \qquad e_i \cdot e_j = \delta_{ij}$$

where scalar product is given by the "low energy" metric, i.e., the metric seen by modes with $k \ll m$:

$$a_k^2 \approx a_o^2 \left[1 + \frac{\beta}{3}\left(\frac{k/a_o}{m}\right)^2\right] = a_o^2 \left[1 + \frac{\beta}{3}\frac{P^2}{m^2}\right]$$

where

$$a_o^2 = \sqrt[3]{\langle \Psi_o | \hat{a}^6 | \Psi_o \rangle}, \qquad \beta := \frac{\langle \Psi_o | \hat{a}^4 | \Psi_o \rangle}{\langle \Psi_o | \hat{a}^6 | \Psi_o \rangle^{\frac{2}{3}}} - 1$$

$P^2 = \delta^{ij} P_i P_j = \delta^{ij} k_i k_j / a_o^2$ is the norm of physical momentum $P_i := e_i^\mu k_\mu$ of mode k.

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Let $E := u^\mu k_\mu = k_0 / N_o$ be the physical energy of mode k. Then the mass shell is

$$-m^2 = g_{(k)}^{\mu\nu} k_\mu k_\nu = -\frac{k_0^2}{N_k^2} + \frac{\sum_i k_i^2}{a_k^2} = -\frac{k_0^2}{N_o^2}\frac{N_o^2}{N_k^2} + \frac{k^2}{a_o^2}\frac{a_o^2}{a_k^2} = -E^2 f^2 + P^2 g^2$$

where the *rainbow functions* are

$$f := \frac{N_o}{N_k}, \qquad g := \frac{a_o}{a_k} = \frac{N_o}{N_k}\frac{a_k^2}{a_o^2} = f\,\frac{a_k^2}{a_o^2}$$

dispersion relation

Following the analysis of Gallego-Torromé, Letizia, Liberati [1507.03205], we use the equation satisfied by a_k to replace $m^2 a_k^6/a_o^6$ with $(\langle\hat{a}^4\rangle k^2 + \langle\hat{a}^6\rangle m^2)/a_o^6 - P^2 a_k^4/a_o^4$:

$$E^2 = \frac{1}{f^2}\left(m^2 + g^2 P^2\right) = \frac{a_k^6}{a_o^6} m^2 + \frac{a_k^4}{a_o^4} P^2 = \frac{\langle\hat{a}^6\rangle}{a_o^6} m^2 + \frac{\langle\hat{a}^4\rangle}{a_o^4} P^2 = m^2 + \frac{\langle\hat{a}^4\rangle}{\langle\hat{a}^6\rangle^{\frac{2}{3}}} P^2$$
$$= m^2 + (1+\beta) P^2$$

where we used $a_o = \sqrt[6]{\langle\hat{a}^6\rangle}$ and $\beta = \langle\hat{a}^4\rangle/\langle\hat{a}^6\rangle^{\frac{2}{3}} - 1$.

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Parameter β encodes the quantum nature of spacetime

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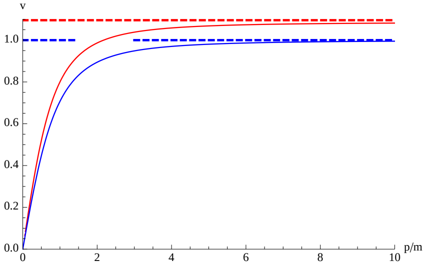
$\Rightarrow$ deformation controlled by parameter β of quantum gravity origin, and amounts to a renormalization of the speed of light: $c_{ren} = c\sqrt{1+\beta}$



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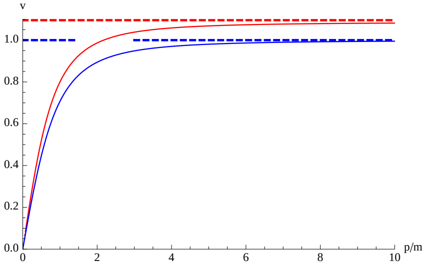

No role of E_{Pl}! For $\beta \approx 0.2$ (red line), large deviations from Lorentz (blue line) already at $P \sim m$ (for protons, $m \ll E_{Pl}$). But can we really detect this? How?

anisotropic cosmologies

Bianchi I metric:

$$ds^2 = -N^2 dt^2 + \sum_i a_i^2 dx_i^2$$

Better use variables $p_1 := a_2 a_3$, and others cyclical.

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2 equations for 4 unknowns N and p_i:

$$N = \sqrt{p_1p_2p_3}, \qquad \sum_i p_i^2 k_i^2 + p_1p_2p_3 m^2 - \gamma(\vec{k}) = 0$$

with

$$\gamma(\vec{k}) := \sum_i \langle \hat{p}_i^2\rangle k_i^2 + \langle \hat{p}_1\hat{p}_2\hat{p}_3\rangle m^2$$

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Luckily, to study dispersion relation, we do not need any complete solution $p_i = p_i(\vec{k})$, but just the low energy limit. Indeed, repeating Liberati's analysis in this case we find

$$E^2 = m^2 + P^2 + \sum_i \beta_i P_i^2, \qquad \beta_i := \frac{\langle \Psi_o | \hat{p}_i^2 | \Psi_o \rangle}{(p_i^o)^2} - 1$$

where $p_i(\vec{k}) = p_i^o + O(\vec{k}/m)$.

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In this limit, the equation becomes

$$p_1^o p_2^o p_3^o = \langle \hat{p}_1 \hat{p}_2 \hat{p}_3 \rangle =: \omega_0$$

which does not uniquely determine p_i^o. However, observing that p_i^o may depend only on ω_0 and $\omega_i = \langle \hat{p}_i \rangle$, we impose the following (arguably reasonable) symmetries:

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- since ω_0 is cyclically symmetric in $\hat{p}$'s, we assume that the three p_i^o depend on ω_0 in the same way
- as a function, p_1^o depends on $\omega_1, \omega_2, \omega_3$ in the same way that p_2^o depends on $\omega_2, \omega_3, \omega_1$, and p_3^o on $\omega_3, \omega_1, \omega_2$

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Hence, we have

$$p_1^o = F(\omega_0, \omega_1, \omega_2, \omega_3), \quad p_2^o = F(\omega_0, \omega_2, \omega_3, \omega_1), \quad p_3^o = F(\omega_0, \omega_3, \omega_1, \omega_2)$$

anisotropic cosmologies

By a dimensional argument it now follows that there are only two possibilities:

1. $p_i^o = \omega_0^{\frac{1}{3}}$
2. $p_1^o = \omega_0^{\frac{1}{3}} \omega_1^{\frac{1}{3}} (\omega_2 \omega_3)^{-\frac{1}{6}}$, and others cyclical

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1. $p_i^o = \omega_0^{\frac{1}{3}}$
2. $p_1^o = \omega_0^{\frac{1}{3}} \omega_1^{\frac{1}{3}} (\omega_2 \omega_3)^{-\frac{1}{6}}$, and others cyclical

Solution 1. leads to $\beta_i = \langle p_i^2 \rangle / \langle \hat{p}_1 \hat{p}_2 \hat{p}_3 \rangle^{\frac{2}{3}} - 1$, which for a semiclassical state Ψ_o gives

$$\beta_i \approx \frac{\langle p_i \rangle^2}{\langle \hat{p}_1 \rangle^{\frac{2}{3}} \langle \hat{p}_2 \rangle^{\frac{2}{3}} \langle \hat{p}_3 \rangle^{\frac{2}{3}}} - 1$$

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Solution 2. leads to the correct classical limit, but all three β_i coincide:

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Remarks:

- isotropic deformation of dispersion relation
- β depends on quantum anisotropies
- consistent with isotropic case, when $\hat{p}_1 |\Psi_o\rangle = \hat{p}_2 |\Psi_o\rangle = \hat{p}_3 |\Psi_o\rangle$

many fields

Consider N non-interacting quantum fields ϕ_A $(A = 1, ..., N)$ not necessarily scalars. The Hamiltonian of the system is

$$\hat{H} = \hat{H}_o + \sum_A \hat{H}_A$$

where $\hat{H}_A$ depends at most on $\hat{a}$, $\hat{p}$, $\hat{\phi}_A$ and $\hat{\pi}_A$.

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The state of the system is $\Psi = \Psi_o \prod_A \varphi_A$. Impose Schroedinger equation as before:

$$\sum_A \left(\prod_{B \neq A} \varphi_B \right) (i\dot{\varphi}_A - \hat{H}^o_A \varphi_A) = 0$$

where $\hat{H}^o_A := \langle \Psi_o | \hat{H}_A | \Psi_o \rangle$.

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Single out φ_1

$$\left(\prod_{B \neq 1} \varphi_B \right) (i\dot{\varphi}_1 - \hat{H}^o_1 \varphi_1) = -\varphi_1 \sum_{A \neq 1} \left(\prod_{B \neq A,1} \varphi_B \right) (i\dot{\varphi}_A - \hat{H}^o_A \varphi_A)$$

and divide both sides by $\prod_B \varphi_B$

$$\frac{1}{\varphi_1}(i\dot{\varphi}_1 - \hat{H}^o_1 \varphi_1) = -\frac{1}{\prod_{B \neq 1} \varphi_B} \sum_{A \neq 1} \left(\prod_{B \neq A,1} \varphi_B \right) (i\dot{\varphi}_A - \hat{H}^o_A \varphi_A)$$

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The equations separate:

$$i\dot{\varphi}_1 - \hat{H}^o_1 \varphi_1 = E_1 \varphi_1, \qquad \sum_{A\neq 1} \left(\prod_{B\neq A,1} \varphi_B \right) (i\dot{\varphi}_A - \hat{H}^o_A \varphi_A) = -E_1 \prod_{B\neq 1} \varphi_B$$

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The first is a Schroedinger equation for φ_1 only

$$i\dot{\varphi}_1 = (\hat{H}^o_1 + E_1\hat{I})\varphi_1$$

The second is of the same type we started with:

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with $\hat{H}^1_A := \hat{H}^o_A - E_1\hat{I}$. We can then repeat the trick, and separate φ_2 and so on. We finally obtain N equations:

$$i\dot{\varphi}_A = (\hat{H}^o_A + E_A\hat{I})\varphi_A$$

E_A represents the effective gravitational potential felt by φ_A due to the presence of the other fields.

many fields

Since back-reaction of matter on geometry is negligible, we set $E_A = 0$. Then the equations decouple:

$$i\dot{\varphi}_A = \hat{H}^o_A \varphi_A \qquad \forall A = 1, ..., N$$

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For every A separately, we compare the equation for ϕ_A with QFT on QST for the same ϕ_A, reading off the algebraic equations and solving them.

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Interpretation: dynamics of N non-interacting quantum fields ϕ_A on quantum spacetime Ψ_o is equivalent to dynamics of the same fields on a 2-parameters family of effective spacetimes

$$ds^2_{A,k} = -N^2_{A,k} d\tau^2 + a^2_{A,k} d\vec{x}^2, \qquad N_{A,k} = a^3_{A,k}$$

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3. physical cosmology
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kinematics

The model: m^2 inflaton minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2\right)$$

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Canonical formulation (details in AD, Lewandowski and Puchta [1302.3038]):

- matter: $\phi \;\longrightarrow\; \phi, \pi$
- geometry: $g_{\mu\nu} \;\longrightarrow\; q_{ab}, p^{ab}$

Constraints: scalar C and vector C_a.

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with $m = 1, 2, ..., 6$ and $k \neq 0$.

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(i) linearization of constraints; (ii) solution of constraints wrt $p_{m,k}$ $(m = 1, ..., 4)$; (iii) gauge-fixing $q_{m,k} = 0$ $(m = 1, ..., 4)$. The only physical degrees of freedom are

- homogeneous isotropic: a, p; ϕ_o, π_o
- "the rest": inflaton ϕ_k, π_k; two polarization of graviton $q_{+,k}, p_{+,k}$ and $q_{\times,k}, p_{\times,k}$

dynamics

Relational time $\phi_o \Rightarrow$ True Hamiltonian of the system

$$\pi_o = \sqrt{C_o(a,p) + \sum_k C_S(a,\phi_k,\pi_k) + \sum_{m=+,\times}\sum_k C_T(a,q_{m,k},p_{m,k})} \approx$$

$$\approx \sqrt{C_o(a,p)} + \sum_k \frac{C_S(a,\phi_k,\pi_k)}{2\sqrt{C_o(a,p)}} + \sum_{k,m=+,\times} \frac{C_T(a,q_{m,k},p_{m,k})}{2\sqrt{C_o(a,p)}} =$$

$$= H_o + \sum_{A=1}^{3} H_A$$

where $A = 1$ for the scalar field ϕ and $A = 2, 3$ for the tensor modes q_m of the metric.

[1] A similar result holds if one uses Mukhanov-Sasaki variables

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$$H_1 = \frac{1}{2H_o}\sum_k \left[\pi_k^2 + (a^4k^2 + a^6m^2)\phi_k^2\right], \qquad H_m = \frac{1}{2H_o}\sum_k \left[p_{m,k}^2 + a^4k^2q_{m,k}^2\right]$$

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Up to H_o^{-1} factor, these are precisely the Hamiltonians of the toy model presented:

- scalar modes behave as a real is massive field
- tensor modes behave as two real massless fields

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result

So we can apply the result for many non-interacting fields, concluding that the effective metric is

$$ds^2_{A,k} = -a^6_{A,k} d\tau^2 + a^2_{A,k} d\vec{x}^2$$

where

$$a_{A,k} = \begin{cases} a(k/m) & \text{if } A = 1 \\ \left[\dfrac{\langle \hat{H}_o^{-\frac{1}{2}} \hat{a}^4 \hat{H}_o^{-\frac{1}{2}} \rangle}{\langle \hat{H}_o^{-1} \rangle} \right]^{\frac{1}{4}} & \text{if } A = 2, 3 \end{cases}$$

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Hence, gravitons move at c, while high energy scalar modes at $c_{ren} = c\sqrt{1+\beta}$.

$\Rightarrow$ We expect Cerenkov radiation

- from scalar particles into gravitons, if $\beta > 0$. Bounds from GRB: $\beta \lesssim 10^{-19}$
- from gravitons into scalar particles, if $\beta < 0$. Bounds from pulsars: $|\beta| \lesssim 10^{-2}$

See Gallego-Torromé, Letizia, Liberati [1507.03205] for details.

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So, how large is β actually? Depends on the quantum gravity theory (for the definition of geometrical operators) and on the choice of state Ψ_o.

loop quantum cosmology

Let $\{|v\rangle\}$ be eigenbasis of volume operator $\hat{v} \sim \hat{a}^3$. The universe today is best described by a Gaussian peaked on large volume $v_o \gg 1$:

$$|\Psi_o\rangle = \frac{1}{N}\int_0^\infty dv\ e^{-[\ln(v)-\ln(v_o)]^2/4s^2}|v\rangle$$

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Then evaluation of β is straightforward:

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where s is the relative dispersion of volume:

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Remarks:

- $\beta < 0$: the dressed speed of light c_{ren} is less than the bare c
- β decays quadratically in $\Delta v/\langle\hat{v}\rangle \ll 1$: tiny QG effect today

GFT quantum cosmology

Let $|\Psi_o\rangle = |\sigma\rangle$ be a GFT condensate concentrated on a single spin j:

$$|\sigma\rangle = \frac{1}{N}\exp\left(\int d^4g\ d\tau\ \sigma(g_1,...,g_4,\tau)\varphi(g_1,...,g_4,\tau)^\dagger\right)|0\rangle$$

with

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It is easy to see that, for any function $f(v(\tau))$ of τ-evolved volume $v(\tau)$, it is

$$\langle\sigma|f(\hat{v}(\tau))|\sigma\rangle = f(V_j)|\sigma_j(\tau)|^2, \qquad V_j \sim \ell^3_{Pl} j^{\frac{3}{2}}$$

which leads to

$$\beta = \frac{\langle\sigma|\hat{v}^{\frac{4}{3}}(\tau)|\sigma\rangle - \langle\sigma|\hat{v}(\tau)^2|\sigma\rangle^{\frac{2}{3}}}{\langle\sigma|\hat{v}(\tau)^2|\sigma\rangle^{\frac{2}{3}}} = \frac{|\sigma_j(\tau)|^2 - |\sigma_j(\tau)|^{4/3}}{|\sigma_j(\tau)|^{4/3}} = |\sigma_j(\tau)|^{2/3} - 1 \sim$$

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Remarks:

- Since j is constant, β grows linearly with the size of the Universe!
- Note that this $|\sigma\rangle$ is not the only candidate for cosmological state in GFT

Outline

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- mechanism for emergence of continuous spacetime $g_{\mu\nu}$ "seen" by the quantum field when it propagates on isotropic quantum cosmology
  - massless scalar field: $g_{\mu\nu}$ is k-indepedent; $E^2 = m^2 + P^2$
  - massive scalar field: $g_{\mu\nu}$ is k-dependent; $E^2 = m^2 + \sqrt{1+\beta}P^2$

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- extension to many non-interacting fields
- application to physical cosmology
  - observational bounds on β
  - estimation of β in LQC (passes the test) and GFT (does not)

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Outlook:

- in cosmology: consider more realistic potentials; test other candidate QG theories
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- geometry sector: generalize the mechanism beyond quantum cosmology
  - ∗ quantum spherical collapse and Black Holes

GRAZIE!