Michal Artymowski, Andrea Dapor, Tomasz Pawlowski

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Analysis

Nonminimally coupled Scalar Field in Loop Quantum Cosmology

Michal Artymowski, Andrea Dapor, Tomasz Pawlowski



Stockholm, 6th July 2012

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Nonminimal Scalar Field

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$$S[g_{\mu\nu},\phi] = \frac{1}{8\pi G} \int d^4x \sqrt{-g} \left[-U(\phi)R + \frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - V(\phi) \right]$$

Nonminimal coupling:

$$U = \frac{1}{2}(1 + \xi \phi^2)$$

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• Reduction to Cosmology: $g_{\mu\nu} \rightarrow a$

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Nonminimal coupling:

$$U = \frac{1}{2}(1 + \xi \phi^2)$$

- Reduction to Cosmology: $g_{\mu\nu} \rightarrow a$
- Canonical analysis: Hamiltonian Constraint

 $H = -6U'\dot{\phi}\dot{a}a^2 - 6U\dot{a}^2a + a^38\pi G\rho = 0$

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Why nonminimal?

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 $\ddot{\phi} + 3H\dot{\phi} = \frac{2U'V - UV' - U'\dot{\phi}^2(3U'' + \frac{1}{2})}{U + 3U'^2}$

Equation of motion for ϕ :

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Equation of motion for ϕ :

$$\ddot{\phi} + 3H\dot{\phi} = \frac{2U'V - UV' - U'\dot{\phi}^2(3U'' + \frac{1}{2})}{U + 3U'^2}$$

Minimal case:

$$\ddot{\phi} + 3H\dot{\phi} = -V' = -m^2\phi$$

 \Rightarrow inflation drived by a fine-tuned heavy field: $\phi_{in} \approx m_{\rm Pl}, m \approx 10^{-6} m_{\rm Pl}$

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Nonminimal case:

$$U(\phi)R + V = \frac{1}{2}R + \frac{1}{2}\phi^2(\xi R + m^2) = \mathcal{L}_{\rm gr} + \frac{1}{2}\phi^2 m_{\rm eff}^2$$

 \Rightarrow Inflation can be driven by a light scalar field.

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Quantization Scheme

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Higgs-based model:

$$V = \frac{\lambda}{4}\phi^4$$

with

 $\xi \approx 47000 \, \sqrt{\lambda}, \qquad \lambda = 0.5$

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Higgs-based model:

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\rightarrow canonical transformation to the minimal form

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Higgs-based model:

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- \rightarrow canonical transformation to the minimal form
- \rightarrow standard loop quantization

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- \rightarrow canonical transformation to the minimal form
- \rightarrow standard loop quantization
- \rightarrow effective limit

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Canonical Transformation

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$$\tilde{g}_{\mu\nu} = 2Ug_{\mu\nu}, \quad \left(\frac{d\tilde{\phi}}{d\phi}\right)^2 = \frac{U+3U'^2}{2U^2}$$

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Canonical Transformation

$$\widetilde{g}_{\mu\nu} = 2Ug_{\mu\nu}, \quad \left(\frac{d\widetilde{\phi}}{d\phi}\right)^2 = \frac{U+3U'^2}{2U^2}$$

For this choice

 $S_{NM}[g_{\mu\nu},\phi]=S_M[\tilde{g}_{\mu\nu},\tilde{\phi}]$

where the potential of field $\tilde{\phi}$ in S_M is given by

$$\tilde{V} = \frac{V}{4U^2}$$

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The "non-minimality" is absorbed in the effective potential \tilde{V} .

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Loop Quantization

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Change to variables

$$\operatorname{sgn}(\tilde{v})\tilde{v} = \frac{\tilde{a}^3}{2\pi\gamma\sqrt{\Delta}\ell_{\rm Pl}^2}, \quad \tilde{b} = -\gamma\sqrt{\Delta}\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}$$

where γ is Barbero-Immirzi parameter and $\Delta = 4\pi\gamma \sqrt{3}\ell_{\text{Pl}}^2$ is "area gap".

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Change to variables

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where γ is Barbero-Immirzi parameter and $\Delta = 4\pi\gamma \sqrt{3}\ell_{\text{Pl}}^2$ is "area gap".

Follow standard LQC:

 $-i\hbar\partial_{\tilde{t}}\Psi(\tilde{v},\tilde{\phi}) = \left[\frac{3\pi G}{8\alpha}\sqrt{|\hat{v}|}\left(\hat{N}^2 - \hat{N}^{-2}\right)^2\sqrt{|\hat{v}|} + \frac{1}{2\alpha}|\hat{v}|^{-1}\hat{\pi}_{\tilde{\phi}}^2 + \frac{\alpha}{\hbar}|\hat{v}|\tilde{V}\right]\Psi(\tilde{v},\tilde{\phi})$ where " $\hat{N} = e^{i\hat{b}/2}$ " and $\alpha = 2\pi\gamma\sqrt{\Delta}\ell_{\rm Pl}^2$.

Loop Quantization

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Effective Limit

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Substitute the fundamental (observable) operators $\hat{\phi}, \hat{\pi}_{\phi}, \hat{\hat{v}}, \hat{\hat{N}}$ with their expectation values:

$$H_{eff} = -\frac{3\pi G}{2\alpha} |\tilde{v}| \sin^2(\tilde{b}) + \frac{\pi_{\tilde{\phi}}^2}{2\alpha |\tilde{v}|} + \frac{\alpha}{\hbar} |\tilde{v}|\tilde{V}$$

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Substitute the fundamental (observable) operators $\hat{\phi}$, $\hat{\pi}_{\phi}$, $\hat{\hat{v}}$, $\hat{\hat{N}}$ with their expectation values:

$$H_{eff} = -\frac{3\pi G}{2\alpha} |\tilde{\nu}| \sin^2(\tilde{b}) + \frac{\pi_{\tilde{\phi}}^2}{2\alpha |\tilde{\nu}|} + \frac{\alpha}{\hbar} |\tilde{\nu}| \tilde{V}$$

The effective dynamics (with respect to \tilde{t}) is obtained from Hamilton equations, with initial conditions

$$\tilde{b} = \pi/2, \quad \tilde{v} = 1, \quad \tilde{\phi} = \tilde{\phi}_{in}, \quad \pi_{\tilde{\phi}} = \pi_{\tilde{\phi}}(\tilde{v}, \tilde{b}, \tilde{\phi})$$

The solutions are parametrized by a single value: $\tilde{\phi}_{in}$.

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Substitute the fundamental (observable) operators $\hat{\phi}$, $\hat{\pi}_{\phi}$, $\hat{\hat{v}}$, $\hat{\hat{N}}$ with their expectation values:

$$H_{eff} = -\frac{3\pi G}{2\alpha} |\tilde{\nu}| \sin^2(\tilde{b}) + \frac{\pi_{\tilde{\phi}}^2}{2\alpha |\tilde{\nu}|} + \frac{\alpha}{\hbar} |\tilde{\nu}| \tilde{V}$$

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The solutions are parametrized by a single value: $\tilde{\phi}_{in}$.

Numerically simulate the evolution, and transform back to non-tilded quantities.

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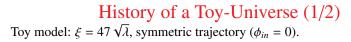
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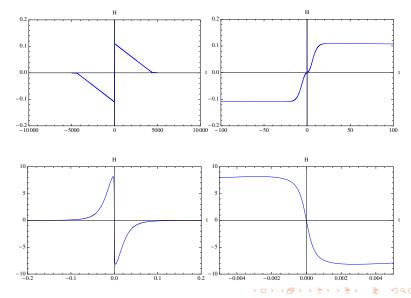
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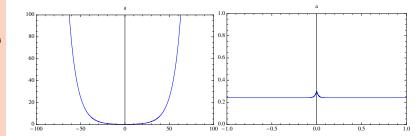
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History of a Toy-Universe (2/2)

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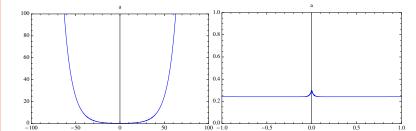
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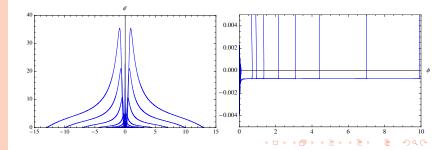
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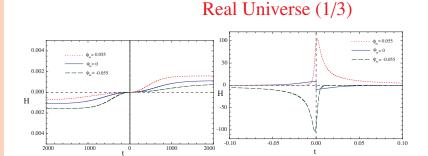


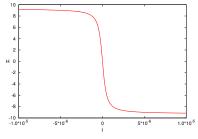
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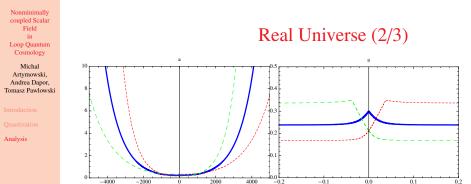
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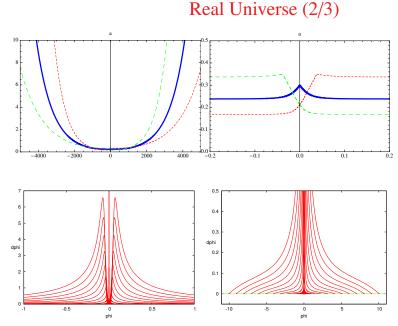


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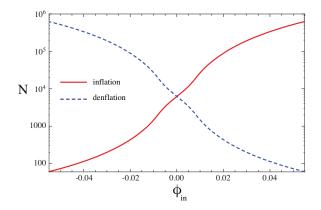


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Real Universe (3/3)

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• The very successful model of non-minimal scalar field admits a simple LQC analogue.

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- The very successful model of non-minimal scalar field admits a simple LQC analogue.
- The LQC-corrected model preserves the usual properties of the classical one: highly probable inflation, driven from light scalar field.

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- The very successful model of non-minimal scalar field admits a simple LQC analogue.
- The LQC-corrected model preserves the usual properties of the classical one: highly probable inflation, driven from light scalar field.
- The LQC-corrected model presents the usual properties of LQC models: resolution of singularity, which is substituted by a (qualitatively new form of) Big Bounce.