Nonminimally coupled Scalar Field in Loop Quantum Cosmology

Michal Artymowski, Andrea Dapor, Tomasz Pawlowski

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Outline

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Nonminimal Scalar Field

$$S[g_{\mu\nu},\phi] = \frac{1}{8\pi G}\int d^4x\,\sqrt{-g}\left[-U(\phi)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Nonminimal coupling:

$$U = \frac{1}{2}(1+\xi\phi^2)$$

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- Reduction to Cosmology: $g_{\mu\nu} \to a$
- Canonical analysis: Hamiltonian Constraint

$$H = -6U'\dot{\phi}\dot{a}a^2 - 6U\dot{a}^2 a + a^3 8\pi G\rho = 0$$

Why nonminimal?

Equation of motion for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} = \frac{2U'V - UV' - U'\dot{\phi}^2(3U'' + \frac{1}{2})}{U + 3U'^2}$$

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Nonminimal case:

$$U(\phi)R + V = \frac{1}{2}R + \frac{1}{2}\phi^2(\xi R + m^2) = \mathcal{L}_{\rm gr} + \frac{1}{2}\phi^2 m_{\rm eff}^2$$

$\Rightarrow$ Inflation can be driven by a light scalar field.

Quantization Scheme

Higgs-based model:

$$V = \frac{\lambda}{4}\phi^4$$

with

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- $\rightarrow$ canonical transformation to the minimal form
- $\rightarrow$ standard loop quantization
- $\rightarrow$ effective limit

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Canonical Transformation

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For this choice

$$S_{NM}[g_{\mu\nu}, \phi] = S_M[\tilde{g}_{\mu\nu}, \tilde{\phi}]$$

where the potential of field $\tilde{\phi}$ in S_M is given by

$$\tilde{V} = \frac{V}{4U^2}$$

The ”non-minimality” is absorbed in the effective potential $\tilde{V}$.

Loop Quantization

Change to variables

$$\mathrm{sgn}(\tilde{v})\tilde{v} = \frac{\tilde{a}^3}{2\pi\gamma\sqrt{\Delta}\ell_{\mathrm{Pl}}^2}, \qquad \tilde{b} = -\gamma\sqrt{\Delta}\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}$$

where γ is Barbero-Immirzi parameter and $\Delta = 4\pi\gamma\sqrt{3}\ell_{\mathrm{Pl}}^2$ is "area gap".

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Follow standard LQC:

$$-i\hbar\partial_{\tilde{t}}\Psi(\tilde{v},\tilde{\phi}) = \left[\frac{3\pi G}{8\alpha}\sqrt{|\hat{\tilde{v}}|}\left(\hat{\tilde{N}}^2 - \hat{\tilde{N}}^{-2}\right)^2\sqrt{|\hat{\tilde{v}}|} + \frac{1}{2\alpha}\widehat{|\tilde{v}|^{-1}}\hat{\pi}_{\tilde{\phi}}^2 + \frac{\alpha}{\hbar}|\hat{\tilde{v}}|\tilde{V}\right]\Psi(\tilde{v},\tilde{\phi})$$

where "$\hat{\tilde{N}} = e^{i\hat{\tilde{b}}/2}$" and $\alpha = 2\pi\gamma\sqrt{\Delta}\ell_{\mathrm{Pl}}^2$.

Effective Limit

Substitute the fundamental (observable) operators $\hat{\tilde{\phi}}, \hat{\pi}_{\tilde{\phi}}, \hat{\tilde{v}}, \hat{\tilde{N}}$ with their expectation values:

$$H_{eff} = -\frac{3\pi G}{2\alpha}|\tilde{v}|\sin^2(\tilde{b}) + \frac{\pi_{\tilde{\phi}}^2}{2\alpha|\tilde{v}|} + \frac{\alpha}{\hbar}|\tilde{v}|\tilde{V}$$

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The effective dynamics (with respect to $\tilde{t}$) is obtained from Hamilton equations, with initial conditions

$$\tilde{b} = \pi/2, \quad \tilde{v} = 1, \quad \tilde{\phi} = \tilde{\phi}_{in}, \quad \pi_{\tilde{\phi}} = \pi_{\tilde{\phi}}(\tilde{v}, \tilde{b}, \tilde{\phi})$$

The solutions are parametrized by a single value: $\tilde{\phi}_{in}$.

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Numerically simulate the evolution, and transform back to non-tilded quantities.

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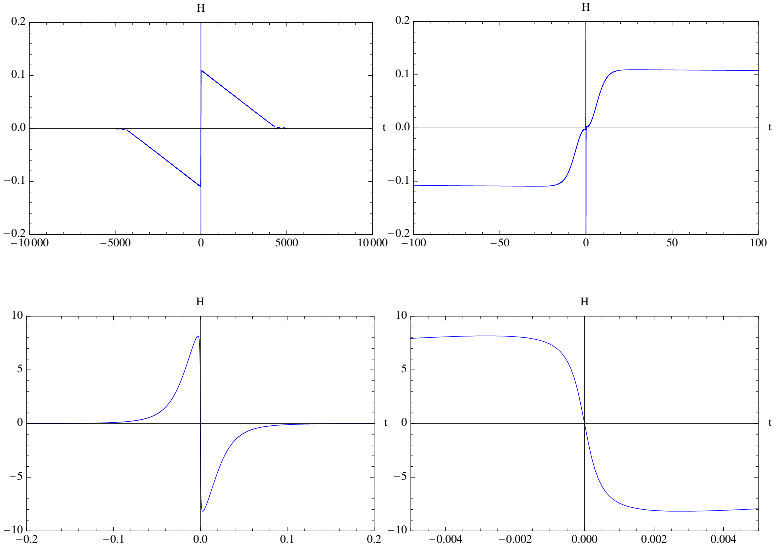
History of a Toy-Universe (1/2)

Toy model: $\xi = 47\sqrt{\lambda}$, symmetric trajectory ($\phi_{in} = 0$).



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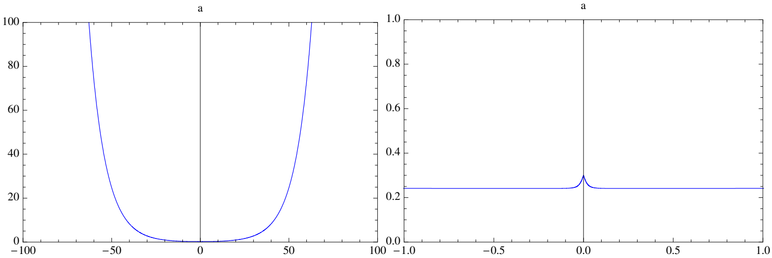
History of a Toy-Universe (2/2)



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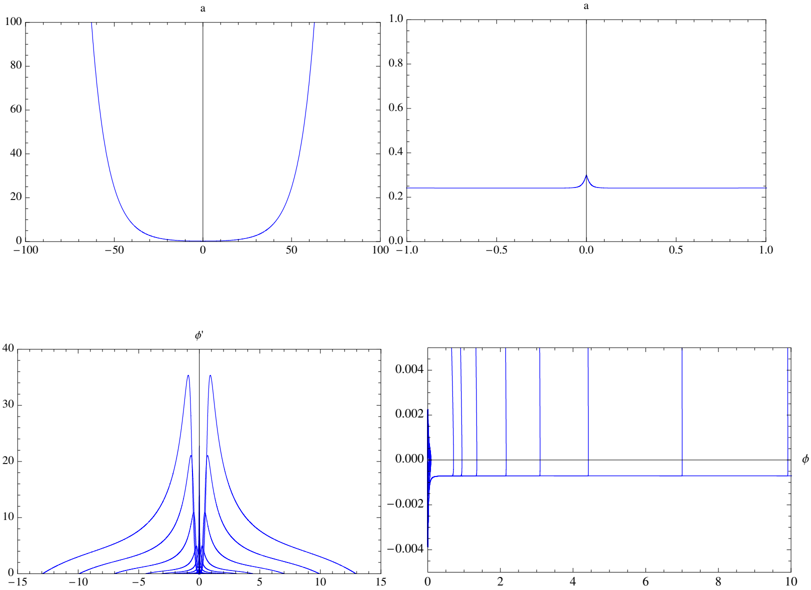
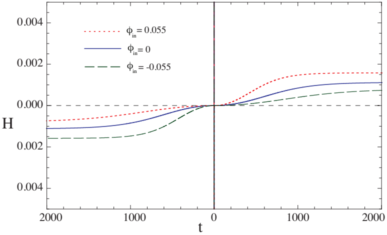
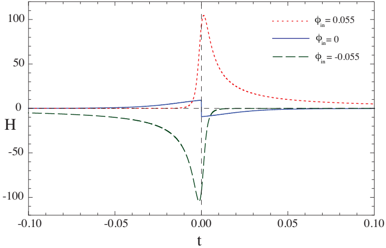
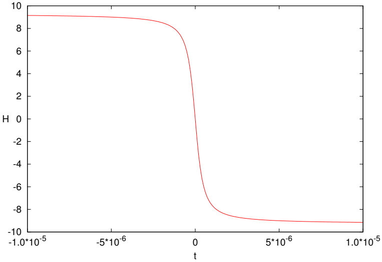


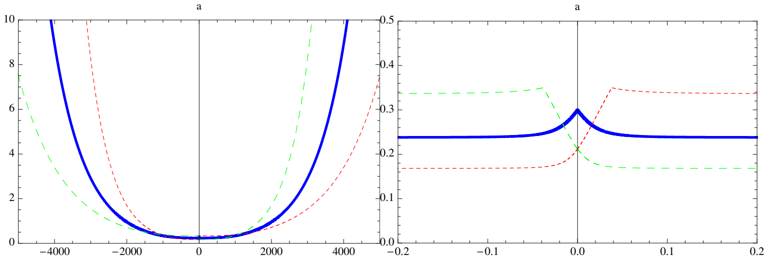
Real Universe (1/3)



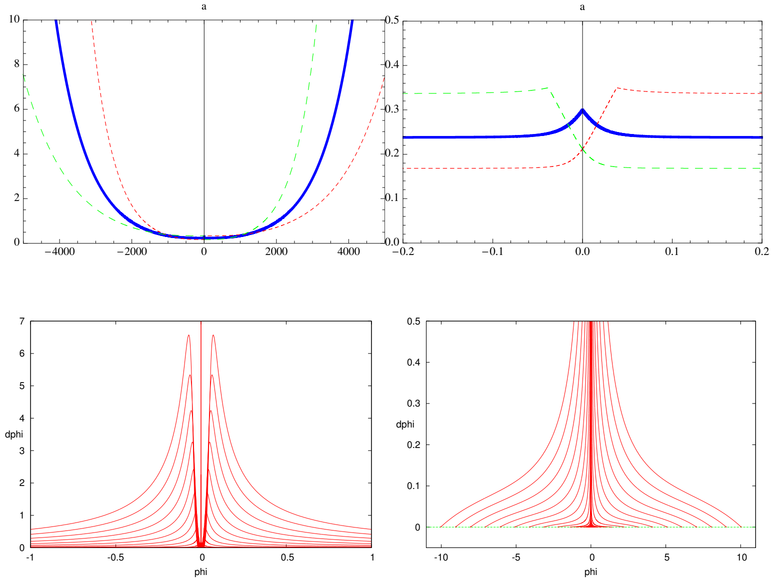




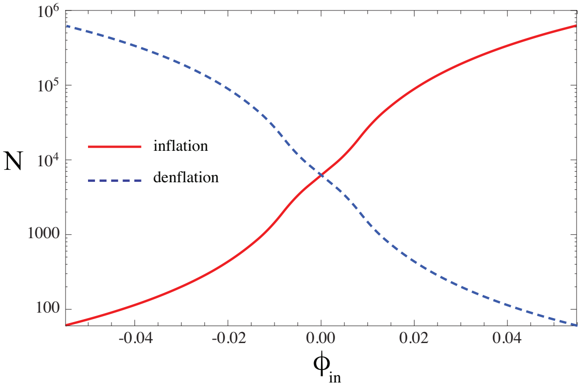
Real Universe (2/3)



Real Universe (2/3)



Real Universe (3/3)



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- The LQC-corrected model preserves the usual properties of the classical one: highly probable inflation, driven from light scalar field.
- The LQC-corrected model presents the usual properties of LQC models: resolution of singularity, which is substituted by a (qualitatively new form of) Big Bounce.