Rainbow metrics and effective cosmological models

Andrea Dapor

Institute for Quantum Gravity, Friedrich-Alexander University Erlangen-Nürnberg

LOOPS'17 – Warsaw, 06 July 2017

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

NATURWISSENSCHAFTLICHE FAKULTÄT

IQG - FAU Erlangen

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

based on

- Ashtekar, Kaminski, Lewandowski – 0901.0933
- Agullo, Ashtekar, Nelson – 1211.1354
- AD, Lewandowski, Puchta – 1302.3038
- Assanioussi, AD, Lewandowski – 1412.6000
- Gallego Torrome, Letizia, Liberati – 1507.03205
- Assanioussi, AD – 1606.09186

Outline

1. an analogy
2. the concept of dressed metric
3. the massive case
4. the case of "many" fields
5. outlook

Phase velocity (velocity of a plane wave of wavevector k)

$$v(k) = \frac{c}{n(k)}$$

with n refractive index.

Relation between $v(k)$ and $\omega(k)$:

$$v(k) = \frac{\omega(k)}{k}$$

So, using $E = \hbar\omega$ and $p = \hbar k$, we find

$$E^2 = \hbar^2 \omega(k)^2 = \hbar^2 k^2 v(k)^2 = c^2 p^2 \frac{1}{n(k)^2}$$

Simple example: $n = (1+\beta)^{-1/2}$, with β independent of k. Then

$$E^2 = c^2 p^2 (1+\beta)$$

Theoretical community: "What is the meaning of β? Where does it come from?"

(1) $$E^2 = c^2 p^2 (1+\beta)$$

Theoretical physicist 1: "Relation (1) is very similar to another relation, the mass-shell, that comes from some very fundamental facts of Minkowski spacetime."

$$E^2 = c^2 p^2 + c^4 m^2$$

Comparison with (1) leads to

$$m = \sqrt{\beta}\frac{p}{c}$$

Theoretical physicist 1: "The spacetime is vacuum Minkowski, while photons have a p-dependent (and possibly t-dependent) mass m."

Theoretical physicist 2: "I don't like that m depends on p. I can derive (1) from mass shell of massless photons on a Robertson-Walker spacetime."

$$ds^2 = -dt^2 + a^2 dx^2$$

implies $0 = g^{\mu\nu} p_\mu p_\nu = -E^2 + c^2 p^2 / a^2$, so

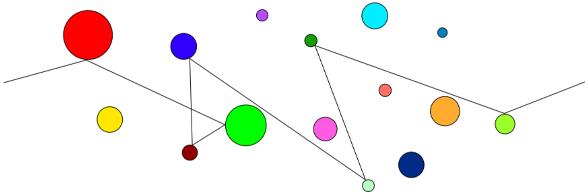
$$E^2 = \frac{1}{a^2} c^2 p^2$$

Comparison with (1) leads to

$$a = (1+\beta)^{-1/2}$$

Theoretical physicist 2: "The spacetime is vacuum Robertson-Walker, while photons are massless."

Who is right? Fundamentally, neither. In reality, photons are massless but space is not empty:



Being slowed down by the interaction, it appears as though they have mass or as though the metric of spacetime is not Minkowski.

However: as far as the phenomenology is concerned, *both* theories are equally good. They explain the result of the experiment and, if no other experiment is available, it is impossible to tell which one is the "right" one.

Outline

1. an analogy
2. the concept of dressed metric
3. the massive case
4. the case of “many” fields
5. outlook

Seminal paper **[Ashtekar, Kaminski, Lewandowski (2009)]**.

Classical Hamiltonian of the system (can be derived from cosmological perturbation theory **[AD, Lewandowski, Puchta (2013)]**):

$$H = H_o + \sum_k H_k$$

where H_o is the function of scale factor a and its conjugated momentum p_a, while

$$H_k = \frac{H_o^{-1}}{2} \left[\pi_k^2 + a^4 k^2 \phi_k^2 \right]$$

A collection of harmonic oscillators with t-dependent frequency.

Formal quantization:

$$\hat{H} = \hat{H}_o \otimes \hat{I} + \frac{1}{2}\sum_k \left[\hat{H}_o^{-1} \otimes \hat{\pi}_k^2 + \widehat{H_o^{-1}a^4}k^2 \otimes \hat{\phi}_k^2\right]$$

acting on Hilberts space of the system $\mathcal{H} = \mathcal{H}_o \otimes \mathcal{H}_M$. (Explicit: hybrid quantization **[Castelló Gomar, Fernández-Méndez, Martin-Benito, Mena Marugán, Olmedo, ...]**.)

Corresponding Schroedinger equation on the state $\Psi \in \mathcal{H}$ of the system:

$$i\frac{d}{dt}\Psi = \hat{H}_o \otimes \hat{I}\Psi + \frac{1}{2}\sum_k \left[\hat{H}_o^{-1} \otimes \hat{\pi}_k^2 + \widehat{H_o^{-1}a^4}k^2 \otimes \hat{\phi}_k^2\right]\Psi$$

Test field approximation: $\Psi = \Psi_o \otimes \varphi$ with $\varphi \in \mathcal{H}_M$ and $\Psi_o \in \mathcal{H}_o$ satisfying

$$i\frac{d}{dt}\Psi_o = \hat{H}_o\Psi_o$$

Strong approximation! Beyond it, see **[Stottmeister, Thiemann (2015)]**.

Schroedinger equation for QFT on QS:

$$(2) \qquad i\frac{d}{dt}\varphi = \frac{1}{2}\sum_k \left[\langle\Psi_o|\hat{H}_o^{-1}|\Psi_o\rangle\hat{\pi}_k^2 + \langle\Psi_o|\widehat{H_o^{-1}a^4}|\Psi_o\rangle k^2\hat{\phi}_k^2\right]\varphi$$

Observation – The same Schroedinger equation is obtained from another system: massless scalar field on a classical R-W geometry $ds^2 = -\bar{N}^2dt^2 + \bar{a}^2\delta_{ij}dx^i dx^j$.

Schroedinger equation for QFT on CS:

$$(3) \qquad i\frac{d}{dt}\varphi = \frac{\bar{N}}{2\bar{a}^3}\sum_k \left[\hat{\pi}_k^2 + \bar{a}^4k^2\hat{\phi}_k^2\right]\varphi$$

Comparing (2) and (3) leads to

$$\langle\hat{H}_o^{-1}\rangle = \frac{\bar{N}}{\bar{a}^3}, \qquad \langle\widehat{H_o^{-1}a^4}\rangle = \bar{N}\bar{a}$$

which admits unique solution

$$\bar{N} = \left[\langle\hat{H}_o^{-1}\rangle^{\frac{1}{3}}\langle\widehat{H_o^{-1}a^4}\rangle\right]^{\frac{3}{4}}, \qquad \bar{a} = \left[\langle\widehat{H_o^{-1}a^4}\rangle/\langle\hat{H}_o^{-1}\rangle\right]^{\frac{1}{4}}$$

Result – Instead of talking of QFT on quantum cosmological spacetime, we can talk of QFT on a classical cosmological spacetime whose line element is

$$ds^2 = -\left[\langle \hat{H}_o^{-1} \rangle^{\frac{1}{3}} \langle \widehat{H_o^{-1} a^4} \rangle\right]^{\frac{3}{2}} dt^2 + \left[\frac{\langle \widehat{H_o^{-1} a^4} \rangle}{\langle \hat{H}_o^{-1} \rangle}\right]^{\frac{1}{2}} \delta_{ij} dx^i dx^j$$

Aside: why doing that?

- we can import techniques from QFT on curved spacetimes
- application to physical cosmology, with modifications to CMB power spectrum **[Agullo, Ashtekar, Nelson (2013)]**

Outline

1. an analogy
2. the concept of dressed metric
3. the massive case
4. the case of "many" fields
5. outlook

Repeat the analysis for massive field. QFT on QS:

$$(4) \qquad i\frac{d}{dt}\varphi = \frac{1}{2}\sum_k \left[\langle \hat{H}_o^{-1}\rangle \hat{\pi}_k^2 + \left(\langle \widehat{H_o^{-1} a^4}\rangle k^2 + \langle \widehat{H_o^{-1} a^6}\rangle m^2\right)\hat{\phi}_k^2\right]\varphi$$

QFT on CS:

$$(5) \qquad i\frac{d}{dt}\varphi = \frac{\bar{N}}{2\bar{a}^3}\sum_k \left[\hat{\pi}_k^2 + \left(\bar{a}^4 k^2 + \bar{a}^6 m^2\right)\hat{\phi}_k^2\right]\varphi$$

Comparing (4) and (5) leads to

$$\langle \hat{H}_o^{-1}\rangle = \frac{\bar{N}}{\bar{a}^3}, \qquad \langle \widehat{H_o^{-1} a^4}\rangle = \bar{N}\bar{a} \qquad \langle \widehat{H_o^{-1} a^6}\rangle = \bar{N}\bar{a}^3$$

which has no solution.

A way out: effective mass $\bar{m}$ different from m. Then

$$\langle \hat{H}_o^{-1}\rangle = \frac{\bar{N}}{\bar{a}^3}, \qquad \langle \widehat{H_o^{-1} a^4}\rangle = \bar{N}\bar{a} \qquad \langle \widehat{H_o^{-1} a^6}\rangle m^2 = \bar{N}\bar{a}^3\bar{m}^2$$

which admits a unique solution

$$\bar{N} = \left[\langle \hat{H}_o^{-1}\rangle^{\frac{1}{3}} \langle \widehat{H_o^{-1} a^4}\rangle\right]^{\frac{3}{4}}, \qquad \bar{a} = \left[\frac{\langle \widehat{H_o^{-1} a^4}\rangle}{\langle \hat{H}_o^{-1}\rangle}\right]^{\frac{1}{4}}, \qquad \bar{m} = m\left[\frac{\langle \hat{H}_o^{-1}\rangle^{\frac{1}{2}} \langle \widehat{H_o^{-1} a^6}\rangle}{\langle \widehat{H_o^{-1} a^4}\rangle^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$

Can we find a solution without dressing the mass? Yes **[Assanioussi, AD, Lewandowski (2014)]**. Recall

$$i\frac{d}{dt}\varphi = \frac{1}{2}\sum_k \left[\langle \hat{H}_o^{-1}\rangle \hat{\pi}_k^2 + \left(\langle \widehat{H_o^{-1}a^4}\rangle k^2 + \langle \widehat{H_o^{-1}a^6}\rangle m^2\right)\hat{\phi}_k^2\right]\varphi$$

$$i\frac{d}{dt}\varphi = \frac{1}{2}\sum_k \left[\frac{\bar{N}}{\bar{a}^3}\hat{\pi}_k^2 + \left(\bar{N}\bar{a}k^2 + \bar{N}\bar{a}^3 m^2\right)\hat{\phi}_k^2\right]\varphi$$

They coincide if

$$\langle \hat{H}_o^{-1}\rangle = \frac{\bar{N}}{\bar{a}^3}, \qquad \langle \widehat{H_o^{-1}a^4}\rangle k^2 + \langle \widehat{H_o^{-1}a^6}\rangle m^2 = \bar{N}\bar{a}k^2 + \bar{N}\bar{a}^3 m^2$$

which admits unique solution (too complicated to write). Crucial difference: $\bar{a}$ not only depends on some expectation values, but also on the ratio k/m.

Result – Instead of talking of QFT on quantum cosmological spacetime, we can talk of QFT on a 1-parameter family of spacetimes:

$$ds_k^2 = -\langle \hat{H}_o^{-1}\rangle^2 \bar{a}_k^6 dt^2 + \bar{a}_k^2 \delta_{ij} dx^i dx^j$$

This is an example of *rainbow metric* **[Amelino-Camelia, Magueijo, Smolin, ...]** breaking local Lorentz-symmetry:

$$E^2 = \frac{1}{f^2}\left(m^2 + g^2 p^2\right)$$

where E and p are the energy and 3-momentum of a particle of wavevector k_μ as measured by an observer u^μ (for which the metric is $\bar{a}_{k=0}$).

Rainbow functions f and g in this case are

$$f = \left(\frac{\bar{a}_0}{\bar{a}_k}\right)^3, \qquad g = \frac{\bar{a}_0}{\bar{a}_k}$$

It was shown **[Gallego Torrome, Letizia, Liberati (2015)]** that the dispersion relation is

$$E^2 = m^2 + (1+\beta)p^2$$

where

$$\beta := \frac{\langle \widehat{H_o^{-1} a^4} \rangle}{\langle \widehat{H_o^{-1} a^6} \rangle^{\frac{2}{3}} \langle \hat{H}_o^{-1} \rangle^{\frac{1}{3}}} - 1$$

Not simply a redefinition of speed of light, because:

- β in general depends on time
- different fields might have different β's

Outline

1. an analogy
2. the concept of dressed metric
3. the massive case
4. the case of "many" fields
5. outlook

Example: 2 fields, one massless and one massive. The state is $\Psi = \Psi_o \otimes \varphi_1 \otimes \varphi_2$ in $\mathcal{H} = \mathcal{H}_o \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$. Repeating the analysis, we find Schroedinger equation

$$i\frac{d}{dt}\varphi_1 \otimes \varphi_2 = \left(\sum_k \hat{H}_{1,k}\varphi_1\right) \otimes \varphi_2 + \varphi_1 \otimes \left(\sum_k \hat{H}_{2,k}\varphi_2\right)$$

with $\hat{H}_{1,k}$ and $\hat{H}_{2,k}$ massless and massive Hamiltonians respectively:

$$\hat{H}_{1,k} = \frac{1}{2}\left[\langle \hat{H}_o^{-1}\rangle \hat{\pi}^2_{1,k} + \langle \widehat{H_o^{-1} a^4}\rangle k^2 \hat{\phi}^2_{1,k}\right]$$

$$\hat{H}_{2,k} = \frac{1}{2}\left[\langle \hat{H}_o^{-1}\rangle \hat{\pi}^2_{2,k} + \left(\langle \widehat{H_o^{-1} a^4}\rangle k^2 + \langle \widehat{H_o^{-1} a^6}\rangle m^2\right) \hat{\phi}^2_{2,k}\right]$$

Putting the same fields on a classical spacetime of Robertson-Walker type, one finds

$$i\frac{d}{dt}\varphi_1 \otimes \varphi_2 = \left(\sum_k \hat{H}'_{1,k}\varphi_1\right) \otimes \varphi_2 + \varphi_1 \otimes \left(\sum_k \hat{H}'_{2,k}\varphi_2\right)$$

where

$$\hat{H}'_{1,k} = \frac{\bar{N}}{2\bar{a}^3}\left[\hat{\pi}^2_{1,k} + \bar{a}^4 k^2 \hat{\phi}^2_{1,k}\right]$$

$$\hat{H}'_{2,k} = \frac{\bar{N}}{2\bar{a}^3}\left[\hat{\pi}^2_{2,k} + \left(\bar{a}^4 k^2 + \bar{a}^6 m^2\right) \hat{\phi}^2_{2,k}\right]$$

Comparison leads to system of equation with unique "bimetric" solution:

$$ds^2_{k,\lambda} = \begin{cases} -\left[\langle \hat{H}_o^{-1}\rangle^{\frac{1}{3}} \langle \widehat{H_o^{-1} a^4}\rangle\right]^{\frac{3}{2}} dt^2 + \left[\dfrac{\langle \widehat{H_o^{-1} a^4}\rangle}{\langle \hat{H}_o^{-1}\rangle}\right]^{\frac{1}{2}} \delta_{ij} dx^i dx^j & \text{if } \lambda = 1 \\ -\langle \hat{H}_o^{-1}\rangle^2 \bar{a}_k^6 dt^2 + \bar{a}_k^2 \delta_{ij} dx^i dx^j & \text{if } \lambda = 2 \end{cases}$$

Result – Each field sees his own metric: for massless field it is a unique metric (no Lorentz-violation), while for massive field it is a rainbow metric.

The β parameter therefore reads

$$\beta = \begin{cases} 0 & \text{for massless particles} \\ \left[\dfrac{\langle \widehat{H_o^{-1} a^4}\rangle^3}{\langle \widehat{H_o^{-1} a^6}\rangle^2 \langle \hat{H}_o^{-1}\rangle}\right]^{\frac{1}{3}} - 1 & \text{for massive particles} \end{cases}$$

Note: $\beta \approx 0$ for semiclassical states $\Rightarrow$ no such deformations today.

But can we estimate the value of β in time, as we approach Big Bang/Bounce?

Experimental bounds on β based on cosmology **[Gallego Torrome, Letizia, Liberati (2015)]** are

$$|\beta| \lesssim \begin{cases} 10^{-15} & \text{if } \beta > 0 \\ 10^{-2} & \text{if } \beta < 0 \end{cases} \tag{6}$$

- LQC **[Ashtekar, Pawlowski, Singh (2006)]**: taking for Ψ_o a gaussian-like state peaked on volume v_o, one finds

$$\beta \approx -\frac{4}{9}\left(\frac{\Delta v}{v_o}\right)^2$$

  $\Rightarrow$ β always negative, and LQC numerics shows it is constant far from bounce and gets smaller at bounce.
  (6) $\Rightarrow \Delta v / v_o \ll 0.15$.
- GFT (single-spin) cosmology **[Gielen, Oriti, Sindoni (2013)]**: I cannot compute β, but $(\Delta v / v_o)^2 \sim 1/\rho(\phi)^2$, which grows as the universe shrinks
  $\Rightarrow$ potential effects on β close to bounce?

Outline

1. an analogy
2. the concept of dressed metric
3. the massive case
4. the case of "many" fields
5. outlook

main message – dressed metrics come with rich phenomenology but many ambiguities

$\Rightarrow$ risky to draw conclusions w/o understanding to what extent the analogy works

Main puzzle: why the two approaches to the massive case produce so different physics?

Meanwhile, "we do what we can":

- anisotropies: Bianchi I shows isotropic deformation of dispersion relation **[Assanioussi, AD (2016)]**
- other symmetry-reduced spacetimes: spherical symmetry **[Gambini, Pullin (2013)]**
- other fields: spinors **[Elizaga Navascués, Martin-Benito, Mena Marugán (2017)]**
- full theory: QRLG w/ scalar field **[Bilski, Alesci, Cianfrani (2015)]**; and soon LQG **[AD, Liegener (2017)]**

Finally, on the relation between dressed metrics and rainbow metrics: recent results on observable effects of deformed dispersion relations.

Thank you for your attention

extraslide 1: explicit form of $\bar{a}_k$

In terms of

$$\delta := \frac{1}{\langle \hat{H}_o^{-1} \rangle} \left[\langle \widehat{H_o^{-1} a^4} \rangle \frac{k^2}{m^2} + \langle \widehat{H_o^{-1} a^6} \rangle \right]$$

it is

$$\bar{a}_k^2 = \begin{cases} u_+ + u_- - \frac{k^2}{3m^2} & \text{if } \frac{4k^6}{27m^6} \leq \delta \\ \frac{2k^2}{3m^2} \cos(\theta) - \frac{k^2}{3m^2} & \text{if } \frac{4k^6}{27m^6} > \delta \end{cases}$$

where

$$u_\pm := \sqrt[3]{\frac{\delta}{2} - \frac{k^6}{27m^6} \pm \sqrt{\frac{\delta^2}{4} - \frac{k^6}{27m^6}\delta}}$$

and

$$\theta := \frac{1}{3} \arccos \left(\frac{27m^6}{2k^6}\delta - 1 \right)$$

extraslide 2: dressed cosmological constant

Divide the 2 fields Schroedinger equation by $\varphi_1\varphi_2$:

$$\frac{1}{\varphi_1}\left(i\frac{d}{dt}\varphi_1\right) - \frac{1}{\varphi_1}\left(\sum_k \hat{H}_{1,k}\varphi_1\right) = -\frac{1}{\varphi_2}\left(i\frac{d}{dt}\varphi_2\right) + \frac{1}{\varphi_2}\left(\sum_k \hat{H}_{2,k}\varphi_2\right)$$

Separation of variables gives

$$i\frac{d}{dt}\varphi_1 = \sum_k \hat{H}_{1,k}\varphi_1 + f(\Psi_o)\varphi_1, \qquad i\frac{d}{dt}\varphi_2 = \sum_k \hat{H}_{2,k}\varphi_2 - f(\Psi_o)\varphi_2$$

where $f(\Psi_o)$ is at most a function of Ψ_o.

These two equations can be obtained from QFT on bimetric Robertson-Walker spacetime (as before), with the addition of "cosmological constants":

$$\Lambda_1 = f(\Psi_o)\sqrt{\frac{\langle \hat{H}_o^{-1}\rangle}{\widehat{\langle H_o^{-1}a^4\rangle^3}}}, \qquad \Lambda_{2,k} = -\frac{f(\Psi_o)}{\bar{N}_k\bar{a}_k^3}$$